Problemas de Polinomios

1 Polinomios

1 Operaciones

1 Dividir con el algoritmo de la división y con Ruffini cuando sea aplicable
Graficar con Geogebra colocando el polinomio en Entrada > Enter. Analizar la gráfica.

$$\frac{2y^3 - 3y^2 + y - 3}{y - 3} \text{ simplify } \rightarrow 3 \cdot y + 2 \cdot y^2 + \frac{27}{y-3} + 10$$

$$\frac{2y^4 - 3y^2 + y - 3}{y^2 - 3} \text{ simplify } \rightarrow 2 \cdot y^2 + \frac{y+6}{y^2 - 3} + 3$$

$$\frac{2y^5 - 3y^2 + 2y - 3}{y^2 + 2y - 1} \text{ parfrac } \rightarrow 10 \cdot y + \frac{66 \cdot y - 30}{y^2 + 2 \cdot y - 1} - 4 \cdot y^2 + 2 \cdot y^3 - 27$$

$$\frac{x^4 - 3 \cdot x^3 - 29 \cdot x^2 + 51 \cdot x + 210}{x^2 - x - 6} \text{ simplify } \rightarrow x^2 - 2 \cdot x - \frac{32}{5 \cdot (x+2)} + \frac{102}{5 \cdot (x-3)} - 25$$

$$\frac{x^4 - 3 \cdot x^3 - 29 \cdot x^2 + 51 \cdot x + 210}{3x - 6} \text{ parfrac } \rightarrow \frac{x^3}{3} - \frac{x^2}{3} - \frac{31 \cdot x}{3} + \frac{188}{3 \cdot (x-2)} - \frac{11}{3}$$

$$\frac{8x^6 + 6x^2 + 1x + 5}{2x - 1} \text{ parfrac } \rightarrow \frac{13 \cdot x}{2^2} + \frac{57}{8 \cdot (2 \cdot x - 1)} + \frac{x^2}{2} + x^3 + 2 \cdot x^4 + 4 \cdot x^5 + \frac{17}{2^3}$$

$$\frac{5 \cdot x^3 - 25 \cdot x^2 + 5 \cdot x + 200}{3x + 7} \text{ parfrac } \rightarrow \frac{5 \cdot x^2}{3} - \frac{305}{27 \cdot (3 \cdot x + 7)} - \frac{110 \cdot x}{3^2} + \frac{815}{3^3}$$

2 Calcular el resto por 3 métodos distintos

$$\frac{2y^4 - 3y^2 + y - 3}{y^2 - y - 6} \text{ parfrac } \rightarrow 2 \cdot y + 2 \cdot y^2 - \frac{3}{y+2} + \frac{27}{y-3} + 11$$

$$\frac{2y^3 - 3y^2 + y - 3}{2y - 3} \text{ simplify } \rightarrow y^2 - \frac{3}{4 \cdot y - 6} + \frac{1}{2}$$

$$\frac{5y^4 + y^2 + 3}{2y + 1} \text{ parfrac } \rightarrow \frac{9 \cdot y}{2^3} + \frac{57}{16 \cdot (2 \cdot y + 1)} - \frac{5 \cdot y^2}{2^2} + \frac{5 \cdot y^3}{2} - \frac{9}{2^4}$$

3 Determinar *a* tal que se verifique la consigna indicada en cada caso
Resolver por el algoritmo de la división, la regla de Ruffini (si es posible) y el Teorema del Resto. Controlar finalmente que la división sea exacta reemplazando el valor de a en el polinomio.

$$\frac{P(x)}{Q(x)} = C(x) + \frac{R(x)}{Q(x)}$$

- $\frac{x^3 - ax}{x^2 - 4}$ Consigna. división exacta.

  R: a = 4

  $\frac{x^3 - 4x}{x^2 - 4}$ parfrac $\rightarrow x$

- $\frac{3x^2 + 2ax - 1}{-3x + 2}$ Consigna. división exacta.

  R: a = $-\frac{1}{4}$

  $\frac{3x^2 - 0.5x - 1}{-3x + 2}$ simplify $\rightarrow -1.0 \cdot x - 0.5$

- $\frac{5x^3 + x^2 - 3ax + 2}{x + 1}$ Consigna. división exacta.

  R: a = $\frac{2}{3}$

  $\frac{5x^3 + x^2 - 2x + 2}{x + 1}$ simplify $\rightarrow 5 \cdot x^2 - 4 \cdot x + 2$

- $\frac{3x^4 - 4ax^3 + x^2 - 1}{x - 2}$ Consigna. división exacta.

  R: a = $\frac{51}{32}$

  $\frac{3x^4 - \frac{204}{32}x^3 + x^2 - 1}{x - 2}$ simplify $\rightarrow 3 \cdot x^3 - \frac{3 \cdot x^2}{8} + \frac{x}{4} + \frac{1}{2}$

- ($5x^2 - 2ax + b$) / (x + 3) Consigna: resto = -4

  R: -7

  $\frac{5x^2 + 14x - 7}{x + 3}$ parfrac $\rightarrow 5 \cdot x - \frac{4}{x + 3} - 1$

- ($4x^3 - 3a^2 x^2 + 3$) / (x – 1) Consigna: resto = 2

  R: $\frac{\sqrt{15}}{3}; -\frac{\sqrt{15}}{3}$

  $\frac{4x^3 - \frac{15}{3}x^2 + 3}{x - 1}$ parfrac $\rightarrow 4 \cdot x^2 - x + \frac{2}{x - 1} - 1$

- $(x^3 - x^2 + ax - 1) / (3x - 2)$ Consigna: resto = -1

  R: $\frac{2}{9}$

  $$\frac{x^3 - x^2 + \frac{2}{9}x - 1}{3x - 2} \text{ parfrac } \rightarrow \frac{x^2}{3} - \frac{1}{3\cdot x - 2} - \frac{x}{3^2}$$

- $\frac{P(x)}{Q(x)} = \frac{8ax^6 + 6x^2 + x + 5}{2x^2 - 2}$ Consigna: x = 2, C(x) = 171

  R: a = 8

  $$\frac{16x^6 + 6x^2 + 1x + 5}{2x^2 - 2} \text{ parfrac } \rightarrow 8\cdot x^2 + 8\cdot x^4 + \frac{7}{x - 1} - \frac{13}{2\cdot(x + 1)} + 11$$

- $\frac{P(x)}{Q(x)} = \frac{2ax^6 + 6x^2 + x + 5}{2x^2 - 2}$ Consigna: x = 3, R(x) = 30

  R: a = 16

  $$\frac{16x^6 + 6x^2 + 1x + 5}{2x^2 - 2} \text{ parfrac } \rightarrow 8\cdot x^2 + 8\cdot x^4 + \frac{7}{x - 1} - \frac{13}{2\cdot(x + 1)} + 11$$

- $P(x) = -3x^3 + x^2 - 2ax + 3$, Consigna: 2 es raíz de P(x)

  R: a= $\frac{31}{4}$

  $$\frac{-3x^3 + x^2 + \frac{17}{2}x + 3}{x - 2} \text{ parfrac } \rightarrow -5\cdot x - 3\cdot x^2 - \frac{3}{2}$$

- $P(x) = ax^2 - 3x - 2a$, Consigna: 1 es raíz de P(x

  R: a= -3

  $$\frac{-3x^2 - 3x + 6}{x - 1} \text{ parfrac } \rightarrow -3\cdot x - 6$$

2 Forma factorial

Factorear, simplificar e indicar el dominio

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$$x^2 + a\cdot x + x + a\cdot x^2 \text{ factor } \rightarrow x\cdot(x+1)\cdot(a+1)$$

$$4\cdot x^3 + 4\cdot x^2 - x - 1 \text{ factor } \rightarrow (2\cdot x+1)\cdot(2\cdot x-1)\cdot(x+1)$$

$$4\cdot(p-2)^4 - (p-2)^2 \text{ factor } \rightarrow (2\cdot p-3)\cdot(2\cdot p-5)\cdot(p-2)^2$$

$$\frac{a^4}{5} - \frac{2\cdot a^3 b}{5} + \frac{a^2 b^2}{5} - \frac{a^3}{5} + \frac{2 a^2 b}{5} - \frac{a\cdot b^2}{5} \text{ factor } \rightarrow \frac{a\cdot(a-1)\cdot(a-b)^2}{5}$$

$$\left(\frac{2x+5}{2x+2} - \frac{x}{x+2}\right)\cdot\left(\frac{2x-5}{2x-4} - \frac{x}{x-2}\right)^{-1} \text{ factor } \rightarrow -\frac{(7\cdot x+10)\cdot(x-2)}{5\cdot(x+2)\cdot(x+1)}$$

$$\frac{x^3 - 4x}{x^3 + 2x^2} \text{ factor } \rightarrow \frac{x-2}{x}$$

$$\frac{4x^2 - 9}{4x^2 + 12x + 9} \text{ factor } \rightarrow \frac{2\cdot x-3}{2\cdot x+3}$$

$$\frac{4x^2 - 1}{2x^3 + x^2} \text{ factor } \rightarrow \frac{2\cdot x-1}{x^2}$$

$$\frac{1-(t-1)^2}{(1+t)^2 - 1} \text{ factor } \rightarrow -\frac{t-2}{t+2}$$

$$\frac{3x}{x-1}\cdot\frac{x^2-1}{x^2+7x} - \frac{2}{x+2}\cdot\frac{x^2-4}{x+7} \text{ factor } \rightarrow 1$$

$$\frac{x+1}{x^2-4} - \frac{x-11}{3x^2-12} \text{ factor } \rightarrow \frac{2\cdot(x+7)}{3\cdot(x-2)\cdot(x+2)}$$

$$\frac{y+2}{1-y^2} - \frac{1}{1-y} + \frac{1}{y+y^2} \text{ factor } \rightarrow -\frac{1}{y\cdot(y-1)\cdot(y+1)}$$

$$\left(\frac{t+1}{t-1} - \frac{t-1}{t+1}\right)\cdot\frac{t^2-1}{2t} \text{ factor } \rightarrow 2$$

$$\frac{5x}{x^2-9} + \frac{2}{x^2+6x+9}\cdot\frac{x+3}{9x} \text{ factor } \rightarrow \frac{45\cdot x^2 + 2\cdot x - 6}{9\cdot x\cdot(x-3)\cdot(x+3)}$$

$$\frac{\frac{1}{x}-\frac{x}{x+1}}{1+\frac{1}{x}} \text{ factor } \rightarrow -\frac{x^2-x-1}{(x+1)^2}$$

$$\frac{-2}{a-2}+\frac{12a-5a^2-4}{a^2-4a+4} \text{ factor } \rightarrow -\frac{5\cdot a}{a-2}$$

$$\frac{x}{x-1}+\frac{3}{x^2-1}-\frac{x^3+3}{x^3-1} \text{ factor } \rightarrow \frac{x\cdot\left(x^2+5\cdot x+1\right)}{(x-1)\cdot(x+1)\cdot\left(x^2+x+1\right)}$$

$$\frac{\frac{4x+8}{x^2-4}}{\frac{(x-2)\cdot(x+3)}{4x+12}} \text{ factor } \rightarrow \frac{16}{(x-2)^2}$$

$$\frac{\frac{x^2+7x+12}{2x^2-32}}{\frac{0.5x+1.5}{x^2-8x+16}} \text{ factor } \rightarrow x-4.0$$

$$p^5-243 \text{ factor } \rightarrow (p-3)\cdot\left(p^4+3\cdot p^3+9\cdot p^2+27\cdot p+81\right)$$

$$100y^4-81a^2p^6 \text{ factor } \rightarrow \left(10\cdot y^2-9\cdot a\cdot p^3\right)\cdot\left(9\cdot a\cdot p^3+10\cdot y^2\right)$$

$$p^3+27 \text{ factor } \rightarrow (p+3)\cdot\left(p^2-3\cdot p+9\right)$$

$$p^3+6p^2+11p+6 \text{ factor } \rightarrow (p+3)\cdot(p+2)\cdot(p+1)$$

$$p^4-5p^2+4 \text{ factor } \rightarrow (p-1)\cdot(p-2)\cdot(p+2)\cdot(p+1)$$

$$4p^3-16p \text{ factor } \rightarrow 4\cdot p\cdot(p-2)\cdot(p+2)$$

$$8p^3-28p^2+2p-7 \text{ factor } \rightarrow (2\cdot p-7)\cdot\left(4\cdot p^2+1\right)$$

$$0.04-\frac{4}{25}y^2+\frac{4}{25}y^4 \text{ factor } \rightarrow \frac{\left(2.0\cdot y^2-1.0\right)^2}{25}$$

$$p^4-7p^3+13p^2+3p-18 \text{ factor } \rightarrow (p+1)\cdot(p-2)\cdot(p-3)^2$$

$$p^3-13p-7p^2+p^4-6 \text{ factor } \rightarrow (p+2)\cdot(p-3)\cdot(p+1)^2$$

$$4-4p-p^2+p^3 \text{ factor } \rightarrow (p-1)\cdot(p-2)\cdot(p+2)$$

$$18p^2-7p^3+8-20p+p^4 \text{ factor } \rightarrow (p-1)\cdot(p-2)^3$$

$p^2 + p^3 - 1 - p \text{ factor} \rightarrow (p-1)\cdot(p+1)^2$

$-2p^2 + p^3 + 8 - 4p \text{ factor} \rightarrow (p+2)\cdot(p-2)^2$

$\frac{1}{p^2}\cdot\frac{p+1}{p^2+p}\cdot\frac{p^4-1}{p^2-1} \text{ factor} \rightarrow \frac{p^2+1}{p^3}$

$\frac{x+\frac{2x}{x-2}}{1+\frac{4}{x^2-4}} \text{ simplify} \rightarrow x+2$

$(x+2)\cdot(x-2) - 2x\cdot(3x-2) + 4 \text{ simplify} \rightarrow -x\cdot(5\cdot x-4)$

$(2-x)\cdot(-3x)\cdot(2+x) + 8 - (2-x)^3 \text{ simplify} \rightarrow 2\cdot x^2\cdot(2\cdot x-3)$

$(5x-1)\cdot 3 + (x-1)(x+1) - (x-1)^2 \text{ simplify} \rightarrow 17\cdot x-5$

$\frac{6t^2+6t+3t^4-3t^3}{t^3+t^2-2t-2} \text{ simplify} \rightarrow \frac{3\cdot t\cdot\left(t^3-t^2+2\cdot t+2\right)}{(t+1)\cdot\left(t^2-2\right)}$

$\frac{\left(a^2-1\right)\cdot\left(9a^2-64\right)}{(3a-8)\cdot(a+1)\cdot(a-1)} \text{ simplify} \rightarrow 3\cdot a+8$

$\left(\frac{1}{y}+\frac{2}{1-y}+\frac{1}{1+y}-\frac{3}{1-y^2}\right)\cdot\left(\frac{1}{y+y^2}\right)^{-1} \text{ simplify} \rightarrow -\frac{1}{y-1}$

$\left(\frac{2x^2+2}{3x^2}-\frac{2x+1}{4x^2-1}\right)\cdot\frac{4x^2-4x+1}{3x}\cdot\left(\frac{x^2+2x+1}{9x^3}\right)^{-1} \text{ simplify} \rightarrow \frac{(2\cdot x-1)\cdot\left(4\cdot x^3-5\cdot x^2+4\cdot x-2\right)}{(x+1)^2}$

$\left[\left(\frac{x}{y}\right)^2-\frac{2x}{y}+1\right]\cdot\left(\frac{3x^2-6xy+3y^2}{y^4}\right)^{-1} \text{ simplify} \rightarrow \frac{y^2\cdot(x-y)^2}{3\cdot\left(x^2+y^2-2\cdot xy\right)}$

$$\sqrt{\frac{x^3 - 3x^2 + 4}{x + 1}} \text{ simplify } \rightarrow \sqrt{(x-2)^2}$$

$$\frac{6t^2 + 6t + 3t^4 - 3t^3}{t^3 + t^2 - 2t - 2} \text{ simplify } \rightarrow \frac{3\cdot t\cdot\left(t^3 - t^2 + 2\cdot t + 2\right)}{(t+1)\cdot\left(t^2 - 2\right)}$$

$$\frac{\left(a^2 - 1\right)\cdot\left(9a^2 - 64\right)}{(3a-8)\cdot(a+1)\cdot(a-1)} \text{ simplify } \rightarrow 3\cdot a + 8$$

3 Identificar

Hallar p y q tal que se verifiquen las siguientes igualdades

$$5x^2 + 2qx^2 - 3x + 5 + p = -2x^2 - qx^2 - 3x - 4 + q$$

$$(0.5q - 3p + 2)\cdot x^3 = -4q\cdot x + 2q\cdot x^2 + 5p\cdot x^3$$

$$\frac{p}{x-2} + \frac{q}{2x} = \frac{3}{x^2 - 2\cdot x}$$

$$\frac{4q}{3(x-5)} - \frac{5p}{x+5} = \frac{2x-1}{x^2 - 25}$$